Partially saturated oscillatory flow under tidal conditions in homogeneous and layered soil columns (experiment and simulations)

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Abstract:
Surface/subsurface flow interactions concern a wide range of applications, from beach morphodynamics (swash zone), coastal aquifers (seawater intrusion and tidal effects), to harbour engineering and hydrology (e.g., man-made structures such as porous dykes and earth dams). The aim of this paper is to study numerically and experimentally the oscillatory flow in a sandy beach under tidal/low frequency forcing. In this short paper, we only describe the principle of the experiments, and then we focus on numerical simulations and analysis. For the numerical simulation, we use Richards's equation for variably saturated/unsaturated flow in a 1D porous column, with oscillatory pressure boundary conditions at the bottom of the column. Numerical simulations and analyses are conducted for the following cases: (1) homogeneous column, (2) heterogeneous 2-layer column (currently being extended to other heterogeneities).

Keywords:

1. Introduction
We focus on oscillatory flows in partially saturated porous media, such as beaches. Our study in this paper concerns 1D vertical flow under tidal forcing, numerically and experimentally. In this short paper, we only describe the principle of the experiments, and then we focus in more detail on numerical simulations and analysis.

Numerical simulations are conducted using the BIGFLOW 3D finite volume flow code that has been widely described, documented and tested (ABABOU & BAGTZOGLOU, 1993). It was used to model oscillatory flows (WANG, 2010). BIGFLOW is based on generalized Darcy and mixed form mass conservation equations. Here, the Van Genuchten/Mualem (VGM) model is used to describe the constitutive relationships \[ \theta(h), K(h) \].

After a brief description of the laboratory experiment, the results of the numerical simulations in a partially saturated 1D porous column under "dynamic" conditions are
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presented. The tidal effect is simulated by an oscillatory "entry pressure head" imposed at the bottom face of the column. It can be expressed as:

\[ h(t) = h_0 + A_0 \sin (\omega_0 t) \]  

(1)

where: "\( h_0 \)" is the positive time-averaged entry pressure head, chosen to coincide with the initial hydrostatic level in the column; "\( A_0 \)" is the amplitude of the pressure wave; \( \omega_0 = 2\pi/T_0 \) is the angular frequency, and \( T_0 \) is the period of the imposed entry pressure. Several periods have been tested, both experimentally and numerically. With respect to the heterogeneity of the soil column, the following cases are studied: (1) homogeneous column (experiment and simulations); (2) two layered column (simulations). The latter work is currently being extended to the case of stratified or multi-layer columns.

2. Laboratory experiment on tidal oscillations in a sand column (summary)

A Darcy-scale laboratory experiment (tide machine) has been designed and constructed at the Fluid Mechanics Institute of Toulouse (IMFT). The system generates low frequency waves (tides) on a partially saturated 1D porous column by applying an oscillatory pressure (simple harmonic function) at the bottom boundary of the column. The setup consists of a sand column, a hydro-mechanical tide generator system or "tide machine", and measurement sensors including TDR probes and porous cup tensiometers, plus a complete data acquisition system operated under LabView. A schematic sketch of the experimental setup is shown in figure 1a, and a photograph of one of the sand columns is shown in figure 1b.

The manufactured hydro-mechanical system is a flexible one that gives the possibility to control and change the mean level, the amplitude and the period of the applied oscillatory pressure.

![Figure 1a. Schematic diagram of the tide machine, soil column and measurement system.](image-url)
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Two views of sand column with tensiometers, TDR probe, and (below) the reservoir.

A small sample of the sand used in the column.

Tide machine: reservoir (left); rotating engine and its supporting structure (right).

Figure 1b. Photographs of the "short column" experiment conducted at the IMFT laboratory (part of a set of tidal flow experiments in sand beaches).

A series of more than 30 experiments on short column (0.55 m height) and long column (1.5 m height) were conducted using this tide machine. For each column, we generate a tidal forcing with different amplitudes $A_0$, frequencies $\omega_0$, and mean water levels $h_0$.

3. Tidal simulations in a homogeneous sand column

We have conducted a parametric study, involving 11 numerical simulations of cyclic flow in the saturated/unsaturated column, with oscillatory pressure imposed at the bottom of the column (see equation 1). A fine sand (FS) is used to illustrate some results in this section. The characteristics of the fine sand are: $K_{\text{SAT}}=1.5\times10^{-4} \text{ m/s}$; $\theta_{\text{SAT}}=0.38$; and for the unsaturated parameters (VGM model): $\alpha=4.6 \text{ m}^{-1}$; $n=5$.

Figure 2 shows the time evolution of the total hydraulic head $H(z,t)=h(z,t)+z$, at different elevations "z" along the column, for an amplitude $A_0=0.10$ m and an average water level $h_0=0.50$ m (relative amplitude $A_0/h_0=0.20$). We clearly see the effects of attenuation (damping) along the column. Other results for the same sand show a phase shift of the pressure signal that increases nonlinearly with "z" (ALASTAL et al., 2010).
Figure 2. (a) Evolution of the total head $H(t)$ at 6 positions along the fine sand column. (b) Water content profiles $\theta(z)$ at 4 different times separated by $T_0/4$.

Parameters of the input pressure signal: $A_0=0.1$ m, $h_0=0.5$ m, $T_0=450$ s.

Figure 3 shows the effect of input parameters ($A_0$, $T_0$, $h_0$) on the amplitude attenuation along the column. We see in figure 3 that the amplitude $A(z)$ of pressure $h(z,t)$ decreases linearly with elevation "z" in the saturated part of the column. This can be partially explained by the linearity of Darcy's head loss law in the saturated zone. In addition, the attenuation rate (the gradient $dA/dz$) increases non-linearly with the mean static level $h_0$.

Other observations: (1) $dA/dz$ increases with $A_0$ (figure 3a), (2) $dA/dz$ decreases with $T_0$ (figure 3b), and (3) $dA/dz$ increases with $h_0$ (figure 3c,d).

4. Tidal simulations in a heterogeneous sand column: two-layer case

We consider now a two-layer column. The bottom layer is a medium sand with $K_{SAT}=2.0 \times 10^{-4}$ m/s, $\theta_{SAT}=0.35$, et $\alpha=11.47$ m$^{-1}$, $n=1.98$. The top layer is a loam (Guelph loam) with $K_{SAT}=3.66 \times 10^{-6}$ m/s, $\theta_{SAT}=0.4$, et $\alpha=1.15$ m$^{-1}$, $n=2.03$. The interface between the two layers is located at elevation $z=57.5$ cm.

The input oscillatory pressure is: $h(0,t)=0.50+0.25 \times \sin(\omega_0 t)$ at $z=0$, so the relative amplitude is quite large here ($A_0/h_0=0.50$).

Figure 4(a) shows the evolution of the total load $H(t)$ at different elevations along the two-layer column: $z=0$ (input signal), $z=30$ cm (always saturated), $z=57$ cm and 58 cm (interface), $z=70$ cm (always unsaturated). The elevation of the free surface $Z_s(t)$ is also plotted: we noted that the column is always saturated from $z=0$ to 35 cm, and always unsaturated for $z\geq65$ cm. The signals $H(t)$ are not harmonic, even in the permanently saturated zone ($z=30$ cm). In addition, we remark that $H(t)$ increases sharply at certain times, which correspond to the crossing of the sand/loam interface by the ascending free surface (this occurs once per cycle).
Figure 4(b) shows the evolution of water contents on both sides of the interface around \(z=57.5\) cm. Periods of saturation are marked in bold. It is noted that the water content varies much more in the medium sand than in the Guelph loam at the interface. The latter is still almost saturated, unlike the sand which drains more easily.

Figure 4(c) shows the water contents profiles \(\theta(z)\) for 8 different times separated by \(T_0/8\). Clearly, water content oscillations are greater in the sand than in the loam layer.

### 5. Conclusions

A series of physical and numerical experiments was conducted on a 1D porous column to study partially saturated oscillatory flow under tidal conditions. Results show the damping and phase shift of the pressure signals along the column. A parametric study quantified the effects of entry pressure parameters on the amplitude damping. Other numerical simulations in a 2-layer column show distinct dynamic behavior in terms of water content for each layer. Other types of heterogeneities are currently under study.
Figure 4. Evolutions in the two-layer column over two cycles. (a) Total head $H(t)$ at different elevations along the column (input signal in dotted curve, and elevation of the free surface $Z_s(t)$ in thick line). (b) Time evolution $\theta(t)$ of volumetric water content of medium sand and Guelph loam on each side of the interface ($z=0.57$ m and 0.58 m). (c) Water content profiles $\theta(z)$ for 8 different times separated by $T_0/8$. Parameters of the input signal at the bottom of the column: $A_0=0.25$ m, $h_0=0.5$ m, $T_0=900$ s.

6. References

