Conférence Méditerranéenne Côtière et Maritime
EDITION 2, TANGER, MAROC (2011)
Coastal and Maritime Mediterranean Conference
Disponible en ligne – http://www.paralia.fr – Available online

Wave interaction with moored floating elastic plate in the presence of end wall

Debabrata KARMAKAR [1], Carlos GUEDES SOARES [1]

1. Centre for Marine Technology and Engineering (CENTEC),
Instituto Superior Técnico, Technical University of Lisbon, Lisboa, Portugal.
guedess@mar.ist.utl.pt

Abstract:
Wave scattering by a finite floating elastic plate connected with mooring lines at its end and with the presence of a wall is analyzed in detail based on the linearized theory of water waves. The solution of the physical problem is obtained using eigenfunction expansion method and by the application of orthogonal mode-coupling relation. The hydroelastic behaviour of the floating elastic plate is investigated by analyzing the effect of the stiffness of the mooring lines on the reflection and transmission characteristics of the gravity waves. The vertical displacement response of the elastic plate are computed and analyzed to understand the effect of mooring on the wave motion below the plate. It is observed that with the increase in the stiffness parameters of the mooring lines the vertical deflections of the floating elastic plate is reduced and with the increase in the distance of the end wall the vertical deflection within the floating elastic plate and the end wall increases. This suggests that the deflection in the transmitted region can be reduced due to the presence of moored floating elastic plate.

Keywords:
Elastic plate – End wall – Mooring lines – Eigenfunction expansion – Amplification factor

1. Introduction

In recent decades, the study on the wave absorption by floating structure near the harbours and narrow channels has gained considerable importance. These floating structures near to the coasts are very important for the reduction of wave height. There has been a lot of work done on the hydroelastic analysis of floating structures by KASHIWAGI (2000), ANDRIANOV & HERMANS (2003) and many other researchers. However, very little work was reported on the wave interaction with floating structures in the presence of an end wall. WU *et al*. (1998) analyzed the wave reflection by a vertical wall with a horizontal submerged porous plate using eigenfunction expansion method. It is observed that in the region close to the wall, there complicated process of wave transformation occur, it includes wave refraction and reflection and leads to wave trapping. In order to prevent the floating structures from being moved away by drift forces, the floating structures are always connected with the

DOI:10.5150/cmcm.2011.013

mooring lines. REN & WANG (1994) investigated the behaviour of a flexible, porous, floating breakwater connected by mooring lines kept under tension by small buoyancy chamber at the tip. It is observed that the hydrodynamic forces and the reflection coefficient increases with the increase in the mooring line stiffness. KHABAKHPASHEVA & KOROBKIN (2002) analyzed the hydroelastic behaviour of the compound floating plates under the influence of surface waves using two different physical approaches to reduce the vibration of the floating structures. In their study, an auxiliary spring-and-mass system was added to reduce the vibration of the main structure.

In the present study, the scattering of surface water waves by a floating elastic plate connected by mooring lines is investigated in the presence of end wall in water of finite depth. The eigenfunction expansion method is used in conjunction with the application of the orthogonal mode coupling relation to obtain the solution for the moored floating elastic plate in the presence of end wall. The effect of the stiffness of the mooring lines on the hydroelastic behavior is investigated by analyzing the amplification factor and plate deflection due to the presence of the end wall.

2. Mathematical Formulation

The problem is analyzed in the two dimensional Cartesian co-ordinate system with the x-axis being taken as horizontal and the y-axis being vertically downward positive with the fluid occupying the region $-\infty < x < \infty$ and $0 < y < h$ as in figure 1.

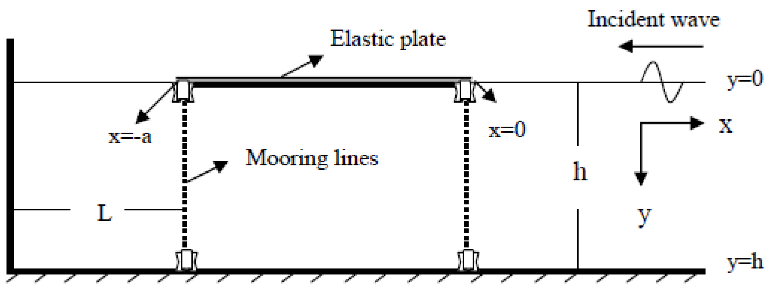


Figure 1. Schematic diagram for floating elastic plate connected with mooring lines.

The elastic plate is modelled under the assumptions of Euler-Bernoulli beam equation. Assuming that the fluid is inviscid, incompressible and the motion is irrotational and simple harmonic in time with angular frequency ω, the velocity potential $\Phi_j(x,y,t)$ and surface elevation $\zeta_j(x,t)$ are expressed in the form $\Phi_j(x,y,t) = \mathrm{Re}\{\phi_j(x,y)e^{-i\omega t}\}$ and $\zeta_j(x,t) = \mathrm{Re}\{\zeta_j(x)e^{-i\omega t}\}$ where Re denotes the real part. The spatial velocity potential $\phi_j(x,y)$ for $j = 1,2,3,$ satisfies the governing Laplace equation in the fluid region is given by:

$$\nabla^2\phi_j(x,y) = 0 \text{ on } -(L+a) < x < \infty,\ 0 < y < h. \quad (1)$$

In the open water region the free surface boundary condition is of the form:

$$\phi_{jy} + \kappa\phi_j = 0, \text{ on } x \in \left(-(L+a), -a\right) \cup \left(0, \infty\right), \; y = 0, \tag{2}$$

where $\kappa = \omega^2/g$ and $j = 1, 3$. Combining the linearized kinematic condition and Euler-Bernoulli beam equation in the presence of compressive force, the linearized plate covered boundary condition is given by:

$$\left(D\partial_x^4 - Q\partial_x^2 + 1\right)\phi_{2y}\left(x, y\right) + K\phi_2\left(x, y\right) = 0, \text{ on } -a < x < 0, \; y = 0, \tag{3}$$

where $D = EI/\left(\rho_w g - m_s\omega^2\right)$, $Q = M/\left(\rho_w g - m_s\omega^2\right)$, $K = \rho_w\omega^2/\left(\rho_w g - m_s\omega^2\right)$, $m_s = \rho_p d$, $EI = Ed^3/[12(1-\nu^2)]$ is the flexural rigidity of the plate, E is the Young's modulus, M is the compressive force, ν is the Poisson's ratio, ρ_w is the density of water, ρ_p is the density of the plate, g is the acceleration due to gravity and d is the draft of the elastic plate. The no flow condition at the rigid bottom for $j = 1, 2, 3$ is of the form:

$$\phi_{jy} = 0 \quad \text{at } y = h. \tag{4}$$

Due to the presence of wall the boundary condition at $x = -\left(L+a\right)$ is given by:

$$\phi_{3x} = 0 \quad \text{at } x = -\left(L+a\right), \quad 0 < y < h. \tag{5}$$

In addition, across the interface between the plates and the free water surface, the continuity of vertical velocity and pressure yields:

$$\phi_{jx}\left(x, y\right) = \phi_{(j+1)x}\left(x, y\right) \text{ and } \phi_j\left(x, y\right) = \phi_{(j+1)}\left(x, y\right) \text{ at } x = 0, -a, \quad 0 < y < h. \tag{6}$$

Assuming that the plates are connected by mooring lines with stiffness q_1 and q_2, at the edges $x = 0, -a$, the bending moment and shear force is related by the relation:

$$EI\phi_{2yxxx}\left(x\pm, y\right) = q_j\phi_{2y}\left(x\pm, y\right) \text{ and } EI\phi_{2yxx}\left(x\pm, y\right) = 0 \quad \text{for } j = 1, 2. \tag{7}$$

It may be noted that if the stiffness constant $q_j = 0$, then the floating elastic plate behaves as a plate with free edge.

3. Method of solution

Using the expansion formulae for wave structure interaction problems, the velocity potentials in each of the regions is given by:

$$\begin{aligned}
\phi_1\left(x, y\right) &= \left(e^{-ik_{10}x} + R_0 e^{ik_{10}x}\right) f_{10}\left(y\right) + \sum_{n=1}^{\infty} R_n e^{-\kappa_{1n}x} f_{1n}\left(y\right) && \text{for } x > 0, \\
\phi_2\left(x, y\right) &= \sum_{n=0,I}^{II}\left(A_n e^{ik_{2n}x} + B_n e^{-ik_{2n}x}\right) f_{2n}\left(y\right) + \sum_{n=1}^{\infty}\left(A_n e^{-\kappa_{2n}x} + B_n e^{\kappa_{2n}x}\right) f_{2n}\left(y\right) && \text{for } x \in (-a, 0), \\
\phi_3\left(x, y\right) &= T_0 \cosh k_{30}\left(x+L+a\right) f_{30}\left(y\right) + \sum_{n=1}^{\infty} T_n \cos\kappa_{3n}\left(x+L+a\right) f_{3n}\left(y\right) && \text{for } x < -a,
\end{aligned} \tag{8}$$

where R_n, $n = 0, 1, 2....$, A_n, B_n $n = 0, I, II, 1, 2....$ and T_n, $n = 0, 1, 2....$ are the unknown constants to be determined. The eigenfunctions $f_{jn}\left(y\right)$ for $j = 1, 2, 3$ are given by:

$$f_{jn}\left(y\right) = \frac{\cosh k_{jn}\left(h-y\right)}{\cosh k_{jn}h} \text{ for } n = 0, I, II \text{ and } f_{jn}\left(y\right) = \frac{\cos\kappa_{jn}\left(h-y\right)}{\cos\kappa_{jn}h} \text{ for } n = 1, 2, 3, ... \tag{9}$$

where k_{jn} for $n = 0, I, II$ are the eigenvalues and satisfies the dispersion relation

$$k_{kn} \tanh k_{jn} h - \kappa = 0 \quad \text{for } j = 1,3, \tag{10a}$$

$$\left(Dk_{jn}^4 - Qk_{jn}^2 + 1\right) k_{jn} \tanh k_{jn} h - K = 0 \quad \text{for } j = 2, \tag{10b}$$

with $k_{jn} = i\kappa_{jn}$ for $n = 1, 2....$ In Eq. (10a) the dispersion relation has one real root k_{j0} and infinite numbers of purely imaginary roots κ_{jn} for $n = 1, 2....$ In Eq. (10b) the dispersion relation has one real root k_{20} and four complex roots k_{2n} for $n = I, II, III, IV$ of the form $\pm\alpha \pm i\beta$. In addition, there are infinite numbers of purely imaginary roots κ_{2n} for $n = 1, 2....$ It may be noted that the eigenfunctions $f_{jn}(y)$ for $j = 1,3$ and $f_{jn}(y)$ for $j = 2$ satisfy the orthogonality relation as given by:

$$\left\langle f_{jm}, f_{jn} \right\rangle_{j=1,3} = \begin{cases} 0 & \text{for } m \neq n, \\ C_n' & \text{for } m = n, \end{cases} \quad \text{and} \quad \left\langle f_{jm}, f_{jn} \right\rangle_{j=2} = \begin{cases} 0 & \text{for } m \neq n, \\ C_n'' & \text{for } m = n, \end{cases} \tag{11}$$

with respect to the orthogonal mode-coupling relation as defined by:

$$\left\langle f_{jm}, f_{jn} \right\rangle_{j=1,3} = \int_0^h f_{jm} f_{jn} dy, \tag{12a}$$

$$\left\langle f_{jm}, f_{jn} \right\rangle_{j=2} = \int_0^h f_{jm} f_{jn} dy - \frac{Q}{K} f_{jm}'(0) f_{jn}'(0) + \frac{D}{K} \left\{ f_{jm}'''(0) f_{jn}'(0) + f_{jm}'(0) f_{jn}'''(0) \right\}, \tag{12b}$$

$$\text{where } C_n' = \frac{2k_{jn}h + \sinh 2k_{jn}h}{4k_{jn} \cosh^2 k_{jn}h} \quad \text{for } j = 1,3, \text{ and } m = n = 0 \tag{13a}$$

$$C_n'' = \frac{2k_{2n}h\left(1 - Qk_{2n}^2 + Dk_{2n}^4\right) + \left(1 - 3Qk_{2n}^2 + 5Dk_{2n}^4\right)\sinh 2k_{2n}h}{4k_{2n}\left(1 - Qk_{2n}^2 + Dk_{2n}^4\right)\cosh^2 k_{2n}h} \quad \text{for } m = n = 0, I, II, \tag{13b}$$

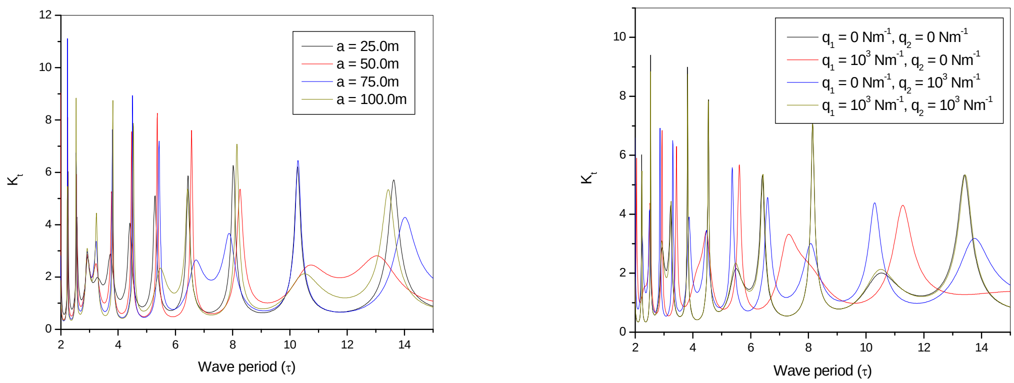
with C_n' and C_n'' for $n = 1, 2, ...$ are obtained by substituting $k_{jn} = i\kappa_{jn}$.

In order to determine the unknown coefficients, the mode-coupling relation (11) is applied on the velocity potential and the eigenfunction along with the continuity of velocity and pressure across the vertical interface $x = 0, -a,\ 0 < y < h$ and the moored edge condition as in Eq. (7) to obtain a system of $(4N + 12)$ linear algebraic equation for the determination of $(4N + 12)$ unknown constants. Once, the unknown constants R_0 and T_0 are determined, the reflection coefficient and the amplification factor are obtained which is given by $K_r = |R_0|$ and $K_t = |T_0|$.

4. Results and discussion

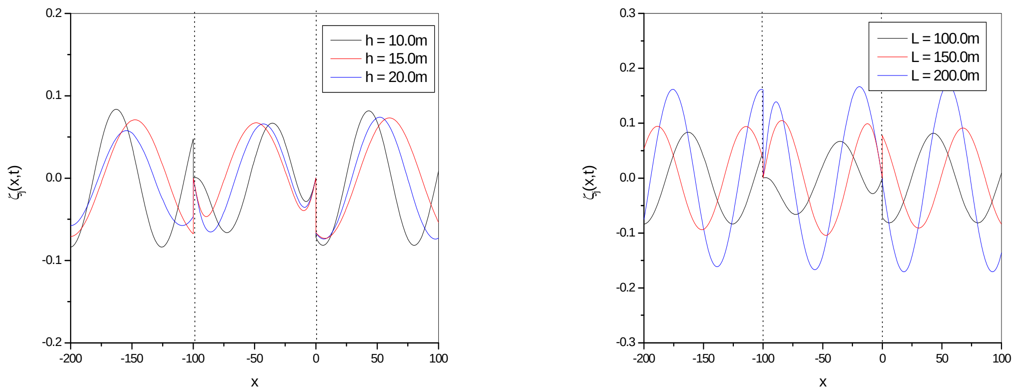
The numerical computations are carried for various cases of stiffness of mooring lines considering $\rho_p / \rho_w = 0.9$, $\nu = 0.3$ and $g = 9.8\ \text{ms}^{-2}$ for different values of the plate length a, plate thickness d, distance of plate from the wall L and water depth h. In figure 2(a) the amplification factor K_t is plotted versus wave period for different values of plate length a with h=10.0 m, L=100.0 m, d=0.25 m considering the mooring stiffness $q_1 = 10^3\ \text{Nm}^{-1}$ and $q_2 = 10^3\ \text{Nm}^{-1}$. It is observed that changing the plate length amplification factor sharply increases for certain values of the wave period. In figure

2(b), the amplification factor K_t is plotted versus wave period for different values of stiffness of the mooing lines q_1 and q_2 with *a*=100.0 m, *L*=100.0 m, *d*=0.25 m and *h*=10.0 m. In this case it is observed that the resonating pattern in the amplification factor is higher for higher values of the stiffness constant and for short wave period.



(a) (b)

Figure 2. Amplification factor K_t versus wave period for different values of (a) plate length a and (b) mooing lines q_1 and q_2.



(a) (b)

Figure 3. Vertical deflection ζ_j versus distance x for different values of (a) water depth h and (b) distance from the wall L.

In figure 3(a), the vertical deflection ζ_j is plotted versus distance x for different values of water depth *h* with *a*=100.0 m, *L*=100.0 m, *d*=0.25 m, *Tp*=7.5 s considering the stiffness constants $q_1 = 10^3$ Nm^{-1} and $q_2 = 10^3$ Nm^{-1}. The vertical deflection in each of the three region is observed to be decreasing with the increase in the water depth. This shows that more waves get transmitted below the plate due to the increase in the water depth. In figure 3(b), the vertical deflection ζ_j is plotted versus distance x for different values of distance of the plate from the wall L with *a*=100.0 m, *h*=100.0 m, *d*=0.25 m, *Tp*=7.5 s considering the stiffness constants $q_1 = 10^3$ Nm^{-1} and $q_2 = 10^3$ Nm^{-1}. It is observed that the vertical deflection keeps on increasing with the increase in the distance of the plate from the wall. This is due to the fact that for smaller *L* the wave reflection is higher and as a result the vertical deflection is reduced but with the increase in distance of the plate from the wall the wave reflection decreases and as a result vertical deflection increases. In both figures 3(a,b), it is seen that near to the edges of the plate x=0 and x=-100, the plate deflection is observed to be zero whereas in the open water region, at the edges it is observed to have jumps or values different from zero. This is due to the application of moored edge condition for the elastic plate and the continuity of velocity and pressure at the edges x=0 and x=-100.

5. Conclusions

An hydroelastic analysis of a moored floating elastic plate in the presence of an end wall is performed in water of finite depth. The detailed analysis of the amplification factor and the vertical deflections are presented for various water depth and plate length. It is observed that the presence of mooring lines at the end of the floating elastic plate reduces the deflection of the floating elastic plate. The presence of an end wall has a great impact on the outgoing wave deflection. It is observed that as the distance of end wall from the floating elastic plate is increased, the deflection between the floating elastic plate and the end wall is significantly reduced. The present study will be helpful to the designers to understand the wave impact on a floating structure in the presence of mooring lines and end wall.

Acknowledgement

This work was performed within the project "FLOATERS–Wave Interaction with Floating Structures in Shallow Water" financed by the Portuguese Foundation for Science and Technology (Fundação para a Ciência e Tecnologia) under contract PTDC/ECM/111242/2009.

6. References

ANDRIANOV A.I., HERMANS A.J. (2003). *The influence of water depth on the hydroelastic response of a very large floating platform*. Marine Structure, 16, pp 355-371. doi:10.1016/S0951-8339(03)00023-6

KASHIWAGI M. (2000). *Research on hydroelastic responses of VLFS: Recent progress and future work*. Intl. J. Offshore and Polar Eng, 10(81), pp 81–90.

KHABAKHPASHEVA T.I., KOROBKIN A.A. (2002). *Hydroelastic behaviour of compound floating plate in waves*. J. Eng. Math. 44, pp 21–40.

REN X., WANG K.H. (1994). *Mooring lines connected to floating porous breakwaters*. Intl. J. Eng. Science, 32(10), pp 1511-1530. doi:10.1016/0020-7225(94)90161-9

WU J., WAN Z., FANG Y. (1998). *Wave reflection by a vertical wall with a horizontal submerged porous plate*. 25(9), Ocean Engineering, pp 767-779. doi:10.1016/S0029-8018(97)00037-1